

## Conservation of Angular Momentum.

In translatory motion, the linear momentum of a single particle is expressed as  $p = mv$ .

Now the question is that what is the analogue of linear momentum in rotational motion? In rotational motion, the analogue of linear momentum is angular momentum.

Consider the case of a particle A having linear momentum  $p$ .

The angular momentum  $L$  of the particle A with respect to fixed point O as origin is defined as

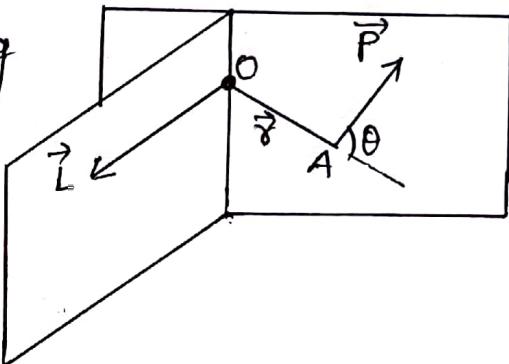
$$L = \boldsymbol{\tau} \times \vec{p} \quad \text{--- (1)}$$

where  $\boldsymbol{\tau}$  is the vector distance of the particle from origin O.

The angular momentum is a vector quantity. Its magnitude is given by

$$L = \tau p \sin \theta \quad \text{--- (2)}$$

Where  $\theta$  is the angle between  $\boldsymbol{\tau}$  and  $\vec{p}$ .  
The direction of  $L$  is perpendicular to the plane formed by  $\boldsymbol{\tau}$  and  $\vec{p}$ .



When a particle moves with angular velocity  $\vec{\omega}$  in a circle, then its angular momentum is given by

$$\begin{aligned} L &= \vec{\tau} \times \vec{P} \\ &= \vec{\tau} \times m\vec{v} \\ &= m(\vec{\tau} \times \vec{v}) \\ &= m[\vec{\tau} \times (\vec{\omega} \times \vec{r})] \end{aligned}$$

$$\text{or, } L = m\vec{r}^2\vec{\omega} \quad \text{--- (3)}$$

Torque:- When a force acts on a particle A, then the moment of the force or torque about O is defined as

$$\vec{\tau} = \vec{\tau} \times \vec{F}$$

We know that

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{d}{dt}(m\vec{v})$$

$$\therefore \vec{\tau} = \vec{\tau} \times \frac{d}{dt}(m\vec{v})$$

$$\vec{\tau} = \frac{d}{dt}(m\vec{v} \times \vec{\tau})$$

$$\text{or, } \vec{\tau} = \frac{d}{dt}L$$

This Torque = rate of change of angular momentum

When  $\vec{\tau} = 0$ ,  $\frac{dL}{dt} = 0$ , or  $L = \text{constant}$

So If the total torque on a particle is zero, the angular momentum of the particle is conserved. This is known as conservation of angular momentum.